OPTIMIZING PLATFORM SURVIVABILITY USING A CHANCE CONSTRAINED LINEAR PROGRAM (U)

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ABSTRACT (U)

(U) On today's battlefield, modern weapons are increasingly able to overmatch the protective capabilities of the armor on today's combat vehicles. The next generation of combat vehicles that the Army procures will need a combination of design strategies yielding a platform well suited to surviving on an increasingly lethal battlefield. Selecting the best design strategies, ones that yield a highly survivable platform, is a problem that the Army needs solved. The goal of this presentation is to demonstrate that optimum design approaches can be obtained as the solutions of a stochastic programming problem.

(U) Discussed is the problem of selecting a "best" set of design options or survivability measures when the performance of these options, as a function to the threat, is known, although the nature of the threat itself is stochastic. A solution to the selection problem is proposed under the assumption that the candidate suites work perfectly. The proposed approach is based on chance-constrained programming.

(U) The chance-constrained programming approach is attractive because it is intuitive and lends itself to a good engineering understanding of the process. Problems with a chance-constrained approach are discussed as well as areas needing additional research.

(U) INTRODUCTION

(U) Background

(U) One of the problems in designing the next generation of weapons platforms is that weapons designers are increasingly able to design weapons whose lethality exceeds the ability of armor to protect the platform. Platform designers can mitigate the effects of weapon lethality by incorporating design changes or active survivability measures of platform protection in addition to advanced armor. These mitigating approaches to platform survivability are called survivability measures.

(U) Survivability measures can include passive design strategies such as “stealthy” platform designs and other methods of lowering platform signature on the battlefield. These measures can also include active methods that search for and detect incoming threats, processing elements to determine the type of threat faced, and some threat defeat mechanism or countermeasure. There are many different concepts to choose from when deciding a suite of survivability measures, and each of these concepts have varying performance levels against each element of the threat array. In this context, concepts include notional systems, survivability components in development, and those survivability measures that have already been fielded.

(U) Choosing a “best” suite of survivability measures from among these various concepts in order to maximize platform survivability when the threat array is known is not difficult, and there are a number of approaches to deciding this “best” suite of survivability measures. However, the threat array is generally unknown or based on projections and expert assessment, which injects randomness into the threat array. The problem becomes much more difficult due to the variability inherent in the threat assessment. The platform designer will never know the exact nature of the threat array that a platform will face on a battlefield due to several factors. Among these factors are the lead time in designing and fielding the platform which implies that the threat array in existence at platform fielding will be estimated or projected based on expert assessment. Other factors include the enemy who will engage the platform, the forces at their command, and even the location of the engagement. All these factors make the platform designer's job difficult.

(U) Problem

(U) When considering a platform's survivability, the current approach is to consider the four areas described in Figure 1. Each "avoidance" is approached differently. For example, a jammer is an active "hit avoidance" survivability measure, which may be used to defeat anti-tank guided missiles. Passive measures such as "penetration avoidance" may be approached by the addition of new or additional armor, and "detection avoidance" is approached through the use of designs minimizing, or masking, platform signatures. These avoidances are employed sequentially, i.e., if "detection avoidance" defeats a threat, then none of the other avoidances are needed. However, in practical situations a combination of survivability measures, addressing each of these avoidances will be needed to achieve the best possible platform survivability against each element of the threat array. For each of these avoidances, there are many survivability options, each with associated burdens (cost, weight, power and volume being common burdens) that are generally not functions of the threat array, and performance levels that are functions of the threat array. The problem then becomes selecting from among these many options the best possible set of survivability measures to maximize platform survivability against the overall threat array while minimizing burden to the platform.

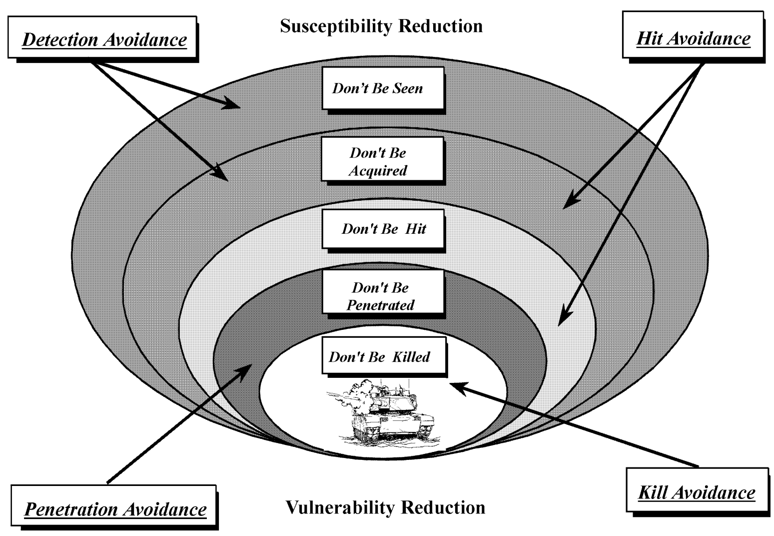

Figure 1: (U) Layered Survivability

(U) PROBLEM STATEMENT

(U) When I began to look at this problem, I made the following observations:

1. The list of survivability measures is discrete, and in a sense complimentary. A designer would likely choose a sensor and a defeat mechanism rather than two sensors or two defeat mechanisms by themselves.

2. This problem can be formulated in stages. If a platform is able to defeat a threat by avoiding detection, then the designer need not consider additional survivability measures for that threat. The same thought applies as you move inward on Figure 1; i.e., once a weapon is defeated no further protection is required. Ideally, the designer would like to defeat a threat as far out on Figure 1 as possible, and

3. The choice of one survivability measure may (or may not) impact the performance of follow on survivability measures, thereby affecting the choice of others; for example, a detection avoidance mechanism may also aid in acquisition avoidance.

(U) The last thing needed is a description of the notation used, and definitions of various probability measures that will be important for this discussion, and that is given in Table I.

Table I. (U) Notation

Variable	Meaning
W	The set of all warning sensors (each element is a single sensor)
C	The set of all countermeasures (each element a single countermeasure)
P	The set of all passive responses (each element is one design option)
P^X	The power set of a set X
A	The set of all active suites and is $\{(w, c) \mid w \in P^W, c \in P^C\}$
S	The set of all survivability suites and is $\{(p, a) \mid p \in P^P, a \in A\}$
Ω	The set of all threats

(U) Two probability measures are used when discussing platform survivability. These measures are the probability of kill and the probability of survival; both these measures are related to Figure 1. The probability of kill is given by:

$$P_k(s, \omega) = \Pi\, P_i(s, \omega) \text{ for } i = 1,2,3,4$$

where

$s \in S$,

$\omega \in \Omega$,.

$P_1(s, \omega)$ = the probability the threat detects the platform,

$P_2(s, \omega)$ = the probability the threat acquires the platform, given that the platform was detected,

$P_3(s, \omega)$ = the probability the threat hits the platform, given platform acquisition, and

$P_4(s, \omega)$ = the probability the threat kills the platform, given a platform hit.

(U) The probability of survival is one of the most important measures of performance a platform designer keys on in the design. The probability of survival for a given platform is given by:

$$P_s(s, \omega) = 1 - P_k(s, \omega).$$

Note: as formulated $P_i(s, \omega)$ may have different values depending on the successful operation of the active survivability measure in s (the a portion of s).

APPROACH

(U) Chance-constrained models relax the feasibility requirement imposed upon a traditional linear program. In a linear program a feasible solution is one that meets all of the constraints. A chance-constrained program allows some, or all of the constraints to be met at a specified level of confidence. Chance-constrained models were first introduced by Charnes and Cooper [ref. 4] and expanded by Van De Panne and Popp [ref. 24], Miller and Wagner [ref. 16] as well as others. Other excellent references include the classic texts by Vajda [ref. 23] and Kall [ref. 13]. Chance-constrained models arise when exact values of some of the problem parameters are not known, such as the nutritive content of cattle feed problem discussed by Van De Panne and Popp.

(U) The selection problem formulated as a chance constrained program is:

$$\min c^T s,$$

$$\text{subject to } A_{dc} s \leq b_{dc}, \text{ and}$$

$$P(P_k(s, \omega) \leq \beta) \geq \alpha \ (0 < \alpha < 1, 0 < \beta < 1),$$

where c is a cost vector, A_{dc} is a matrix of purely deterministic design criterion, b_{dc} is a vector of deterministic design goals, $s \in S$, $\omega \in \Omega$, and the final constraint is the chance constraint. For the selection problem at hand, this constraint represents the probability that the threat succeeds in killing the target, $P_k(s, \omega)$, is less than some desired level β, and this constraint is met with a probability α. An equivalent representation of the chance constraint base on survivability yields the following:

$$P(P_s(s, \omega) \geq 1 - \beta) \geq \alpha).$$

where α and β are defined as above.

(U) Miller and Wagner indicated that if the following condition was true, a log transform of the product in the chance constraint allows replacement of the probability product with a sum of the log probabilities [ref. 16].

$$-d \ln P_s(s, \omega) / d\omega = p_k(s, \omega) / [1 - P_k(s, \omega)] \equiv q(s, \omega), \text{ for } P_k(s, \omega) < 1,$$

where $q(s, \omega)$ is called the hazard rate, s and ω are defined as above. If $q(s, \omega)$ is an increasing function, then $\ln P_s(s, \omega)$ is concave, and therefore the product constraint is defined on a convex set. Taking the log of the chance constraint allows us to place the selection problem in a form that is solvable by linear programming methods. This yields:

$$F(s, \omega) = - \ln \beta + \Sigma \ln P_i(s, \omega) \text{ for } i = 1, 2, 3, 4.$$

(U) In addition to Miller and Wager, several others have presented related work regarding log-concavity and convex measures. Van De Panne and Popp [ref. 24] established the conditions under which the chance constraint is a convex constraint for joint normal distributions. Prekopa's work in [ref. 19] is widely regarded as an important generalization of log concave measures in stochastic programming. Prekopa established the general conditions under which the feasible set for a chance constraint is a convex set. Vajda notes that, while many commonly used distributions are not convex (e.g., normal and uniform), convex programming techniques can be used if by taking the log we obtain a sum of concave functions [ref. 23]. Finner and Roters [ref. 11] have studied log-concavity for the Chi-square, F and Beta distributions.

(U) Solution Methods

(U) Solution methods for chance constrained linear programs fall are either exact or approximate. Little has been written about exact solutions given the need to know the underlying distributions, but much has been written on approximate solutions to chance constrained linear programs. Each of these approaches will be discussed in this section.

(U) When the distributions and parameters characterizing them (means and covariances) are known, solving chance-constrained programs is done by replacing probabilistic constraint with a deterministic equivalent based on these parameters. Vajda [ref. 23], Kall [ref. 13], Mitra [ref. 17], and Sposito [ref. 21] are good references for this depending on the nature of the problem. Szantai [ref. 22] presents a computer code for solving these problems when the resulting distributions are normal. Prekopa discusses several approaches to solving chance-constrained models in [ref. 20] including gradient methods and the use of penalty functions. Eisner and Olson discuss the use of duality in solving chance constrained models [ref. 8].

(U) While there are methods of solving chance-constrained programs exactly, there are many practical cases in which it is not possible to solve the system in an exact form. Many people have suggested approximate solutions, which can be grouped into three categories. The first and most extensive category is that based on interval or bounding techniques. The second category involves replacing the chance constraint by a linear approximation, and the final category is a relaxation technique. Kall, Ruszczynski and Frauendorfer survey of various approximation approaches in [ref. 15], discussing worst case analysis, discrete approximations, bounding, and other techniques.

(U) Among the interval or bounding techniques, Hillier [ref. 12] presents both an exact and an approximate solution when the decision variables are zero/one constrained or continuous. Hillier's technique involves placing upper and lower bounds on the chance constraint, where these bounds are estimated or calculated from the parameters of the distributions. Hillier's method seems well suited for use when the number of points needed to estimate the bounds is small. Allen, Braswell and Rao [ref. 2] discuss a method that does not depend on the distribution, but on a tolerance region and sample. These authors also show how the chance constraint set can be transformed into a distribution free set of constraints. Dupacová discusses an approach based on worst case, or expert assessment of the resulting distribution, and then he uses these to approximate the chance constraints [ref. 7].

(U) Linear approximations of the chance constraints are the second major category of approximation approaches. In all, I've selected four papers as representatives of this approach. In [ref. 5] Charnes, Kirby and Raike introduce the concept of an acceptance region. They use the concept of an acceptance region to establish the presence or absence of an optimal solution. When an optimal solution exists, they show that there is an optimal piecewise linear decision rule that will find it. Olson and Swenseth present an approach based on a penalty function [ref. 18]. The authors use a linear approximation to the chance constraint, and then impose the penalty function, which as at least as great as the chance constraint. The advantage to this approach is that it is readily adaptable to joint distributions which are not independent. Olson and Swenseth also present the results of this linearization as they applied it to the problem Van De Panne and Popp solved in [ref. 24] with very good agreement. Akinfiev and Zharov present an approximation approach based on deterministic equivalents and apply that approach to a planning problem [ref. 1]. The final paper by Elsayed and Ettouney [ref. 9] replaces the chance constraint with one based on a Taylor series expansion and then solves the resulting program recursively to a desired tolerance level. Elsayed and Ettouney also apply the Taylor series expansion to two examples and present the resulting solutions.

(U) The final approximate solution category contains a paper by Charnes, Chang and Semple [ref. 3]. In this paper, the authors discuss the concept of relaxation applied to the chance constraint. They propose a method of relaxing a set of joint constraints into a semi-infinite set of individual constraints. The paper also develops tight relaxation to constraints where only partial distribution (mean and variance) information is known. The authors also apply this technique to examples provided by Prekopa and Szantai with good agreement.

(U) Within the three categories of approximations above, five papers present approaches that should be applicable to solving the selection problem. The papers are those by Hillier [ref. 12], Charnes, Kirby and Raike [ref. 5], Olson and Swenseth [ref. 18], Elsayed and Ettouney [ref. 9], and by Charnes, Chang and Semple [ref. 3]. All of the papers present approaches that appear, with little modification, to be readily adaptable to problems with dependence or conditional distributions in addition to being readily useable for problems with independent joint distributions.

(U) SUMMARY

(U) Weapons designers are increasingly able to field weapons whose lethality easily overmatches the armor on both current and next generation weapons platforms. Platform designers can mitigate the effects of weapon lethality by incorporating design changes or active survivability measures of platform protection in addition to advanced armor. These mitigating approaches to platform survivability are called survivability measures, and the set of measures applied to a platform is called a survivability suite.

(U) The recognition that a designer may not be able to achieve 100 percent survivability against all threats suggests a situation where relaxing the survivability criteria allows one to achieve a “optimum” solution. This is chance programming. The possibility of inter-relationships among some combinations of survivability measures in a suite (either a mutual degradation or a synergy where the whole is greater than the sum of the parts) suggests approaches that allow interactions to occur. That is the incorporation of a covariance matrix into the analysis process.

(U) When the designer makes the assumption a selected survivability suite works probabilistically, and that 100% survivability is unattainable, the selection problem lends itself to a chance programming approach. The chance programming approach, from an engineering perspective, is intuitive. The deterministic constraints are ones that the designer can know or estimate very well, and the design goals are ones that are easily understood. The objective function, in this sense, is not important if the designer leaves the coefficients set to unity. However, by changing the weights of the objective function, the designer can create preferences for certain solutions. For example, notional designs, which may not be the preferred course of action, may have a “cost” higher than survivability designs already completed. Only if the notional design is considerably better than what is already completed will the notional design be chosen. Again, this is an intuitive feature. And finally, the chance constraint is easily understood from a systems engineering viewpoint.

(U) Several problems should be noted. The first problem is actually the assumption that the survivability suite works properly. In a real world situation this is not the case, but the assumption is a good one for establishing an upper bound on anticipated performance levels. Furthermore, this assumption allows a very good first cut at solving the selection problem.

(U) The second problem relates to the first problem in the sense that to truly optimize survivability, the performance of the suite should be modeled as well. For example, there is a small, but non-zero probability that a threat classifier may miss-identify a threat. A survivability suite that is a robust performer against a wide array of threats should account for the functioning of the survivability components as well.

(U) The final problem is that this while approach may lend itself to selecting a “best suite,” it may miss solutions where a combination of “active” (active sensor/countermeasure combination) and “passive” approaches may yield better survivability (additional armor and signature minimizing designs). Coupling both approaches may yield better survivability against a broader class of threats, but the chance programming approach seems to lend itself to one or the other type of approach, but generally not both.

(U) CONCLUSION

(U) Selecting the components of a survivability suite is a problem in stochastic programming and it has the potential for research in a number of different areas. The three problems identified in the summary all present a degree of difficulty and all of them will require a good deal of study to solve. The selection problem, cast as a chance constrained program, is a good first step in defining a method for selecting a best set of survivability options.

(U) The Army survivability community needs a tool that can optimize a platform’s survivability against a broad array of threats. A method that can determine a “best” suite of survivability measures is something that can benefit the Army’s survivability community as a whole. The selection of this “best” set of survivability measures is a stochastic programming problem, and the first step in creating the methodology is by exploring chance programming.

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